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Journal of Fluids and Structures 26 (2010) 602–610

JOURNAL OF FLUIDS AND STRUCTURES

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Vibration frequency and lock-in bandwidth of tensioned, flexible cylinders experiencing vortex shedding

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Received 8 October 2008; accepted 15 February 2010
Available online 20 March 2010

Abstract

In-water vortex-induced vibration (VIV) tests of top-tensioned, flexible cylindrical structures were conducted at Shell Westhollow Technology Center current tank. These tests revealed that the top tension and structural stiffness (both lateral and axial) can have a significant impact on vibration frequencies. During lock-in between the vortex-shedding frequency and the structure's natural frequency, the increase of the vibration frequency with flow speeds is strongly related to the rise of the axial tension. After an initial abrupt rise, the vibration frequency of a bending-stiffness-dominated structure only increased slightly during lock-in. Alternative explanations are provided on why the vibration frequency does not rise significantly but there can still exist a broad lock-in band, and why a more massive structure has a narrower lock-in bandwidth.

Keywords: Vortex-shedding; Vibration frequency; Flexible cylinder; Lock-in band; Tension; Stiffness

1. Introduction

VIV of subsea pipelines and marine risers, used in offshore oil and gas exploration and production, are a practical concern in design. If not properly protected from VIV, these structures may fail due to oscillating stress-caused fatigue, at frequencies often higher than those resulting from ocean waves, in a very short period of time.

VIV has been a subject of intensive research. These efforts have been focused mostly on the effects of fluid-related parameters, such as Reynolds number (Lienhard, 1966), flow profile (Vandiver et al., 1996), mass ratio (Sarpkaya, 1979; Vandiver, 1993; Govardhan and Williamson, 2002), hydrodynamic damping (Griffin et al., 1975), and surface roughness (Allen and Henning, 2001; Achenbach, 1971). Investigations into structural effects on VIV of a flexible cylinder, most prominently the tension, lateral and axial stiffness, are scarce. A systematic study of these parameters, to the authors' knowledge, does not exist. Huse et al. (1998) studied experimentally a flexible riser model with top tension and concluded that the dynamic axial stresses due to VIV should be considered in design. Vandiver (1993) investigated the dynamic effects of VIV on finite and infinitely long structures and developed parameters that govern such response behavior. Gharib (1999) studied experimentally the spring constants of a spring mounted rigid cylinder. Sparks (2001) performed studies to examine riser VIV and provided simplified analytical solutions for such vibrations, which

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0889-9746/$ - see front matter
doi:10.1016/j.jfluidstructs.2010.02.002

Nomenclature

D	cylinder outside diameter	I	area moment of inertia
E	Young's modulus	L	cylinder length
f_1	in-water natural frequency of a cylinder in its first bending mode, in Hz	m	mass per unit length of a cylinder
		T	tension
		V	water speed
		V_r	reduced velocity, given by V/f_1D

compared well with finite element-based models. Lee and Allen (2009) questioned the theoretical basis of the Griffin plot for VIV motion prediction, since it does not account for the effects of structural stiffness.

Vibration frequencies and lock-in bandwidth are both important pieces of information for VIV design. The vibration frequency is directly related to the fatigue damage from VIV. A structure vibrating at a higher frequency will fail in a shorter time. The lock-in bandwidth is used by designers to determine the potentially harmful current speeds. A broader band signals a wider range of critical speeds. It is widely held that during lock-in the added mass decreases with the flow speed, and that makes the natural frequency and thus the vibration frequency rise. This is the reason why there exists a broader lock-in zone for lighter structures since the added mass effects are more pronounced to these cylinders than to the heavier ones (Vandiver, 1993).

In-water, VIV tests of a number of top-tensioned, flexible cylindrical structures were conducted at Shell Westhollow Technology Center current tank. Two such structures, with outside diameters of 2.5 and 4.5 in. (1 in. = 2.54 cm), were selected to investigate the effects of the top tension, lateral and axial stiffness on VIV responses, in particular the vibration frequency and lock-in bandwidth. These tests revealed some facts which are in contradiction to our common beliefs, and new insights are gained by carefully studying the test data. In the following sections, the test facility and setup are first described, and test results are then presented and discussed, followed by conclusions and recommendations.

2. Test description

2.1. Current tank facility

The Shell Westhollow current tank steel test structure is built to circulate (fresh) water at current speeds of 0–2.13 m/s. A ship's propeller driven by a hydraulic power package circulates the water. Two honeycomb sections (straighteners) are used to minimize turbulence and fluid rotational effects, and a shear screen can be used to produce sheared velocity profiles when desired. A 15.24 m deep, 0.91 m inside diameter steel caisson is located in the test-section to allow for test cylinders as long as about 18 m. The excitation region of the test-section is 3.66 m deep by 1.07 m wide and is produced by a fixed steel insert with baffles that change the cross-sectional dimensions of the flow from 2.13 m deep by 1.83 m wide to 3.66 m deep by 1.07 m wide, and then back to 2.13 m deep by 1.83 m wide beyond the test-section. A plan view and an elevation view of the tank test-section are displayed in Fig. 1.

2.2. Test set-up and test parameters

The two test cylinders were both Acrylonitrile Butadiene Styrene (ABS) tubes (see Table 1 for their properties), with the same length, 3.72 m. The cylinders were terminated at lower and upper ends by universal joints (Fig. 2). These ball joints were designed such that they allowed motion in all directions except torsional motion. A biaxial accelerometer was mounted inside the pipe at the center of the cylinders. A bending load cell, made by our own staff, was mounted beneath the lower ball joint to monitor loads in the in-line and cross-flow directions.

Similarly, a bending load cell was mounted above the upper ball joint inside a sleeve. A tension load cell was placed on top of a spring that was linked at the top of the bending load cell. A rod was connected at the top of the tension load cell. A tension was applied at the top for all test cases. The lower end was constrained such that all translational movements were restricted. At the upper end, the pipe was allowed to move vertically (resisted only by the spring), but not laterally. The center-to-center distance between the ball joints was approximately 3.73 m. The top end (ball joint) was not submerged (about 0.3 m above the still waterline).

Also listed in Table 1 are test speed range and speed increment for each cylinder. The mass ratio is the mass of the pipe, including that of the contents, divided by the mass of the displaced water. The in-water natural frequencies are also provided. The first natural frequencies were obtained from the pluck tests in which an initial displacement was

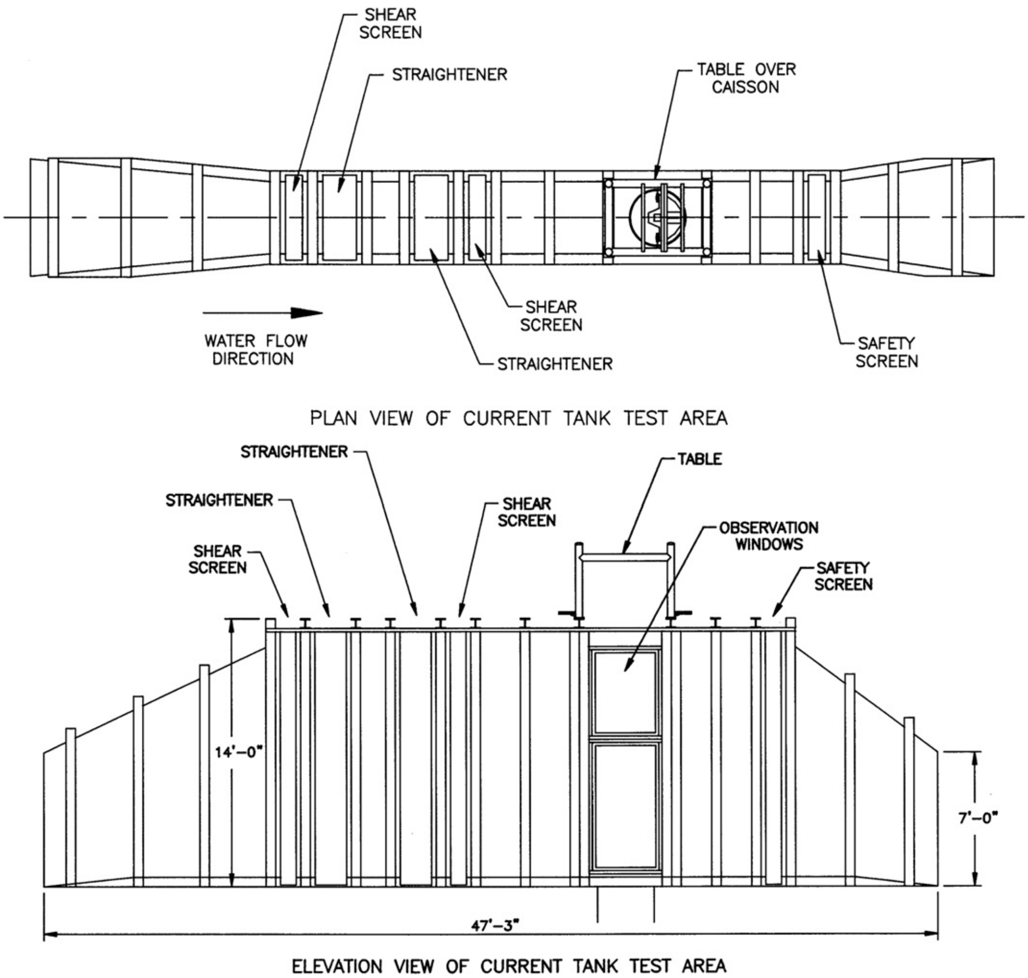

Fig. 1. Current tank test area plan and elevation view (1 ft = 0.3048 m).

Table 1
ABS test cylinder properties and test parameters.

Test cylinder	**4.5 in.**	**2.5 in.**
Outside diameter, D (mm)	114.7	63.5
Inside diameter (mm)	97.0	50.8
Length, L (m)	3.72	3.72
Aspect ratio, L/D	32	59
Pipe air weight (N/m)	28.5	11.2
Content weight (N/m)	72.3	19.9
Minimum test speed (m/s)	0.762	0.305
Averaged speed increment (m/s)	0.060	0.076
Maximum test speed (m/s)	2.134	1.676
First in-water frequency (Hz)	2.30	1.72
Second in-water frequency (Hz)	9.00	6.31
Pretension, T (N)	222.4	222.4
Bending rigidity, EI (N m^2)	8497.0	963.6
Mass ratio	0.996	1.0

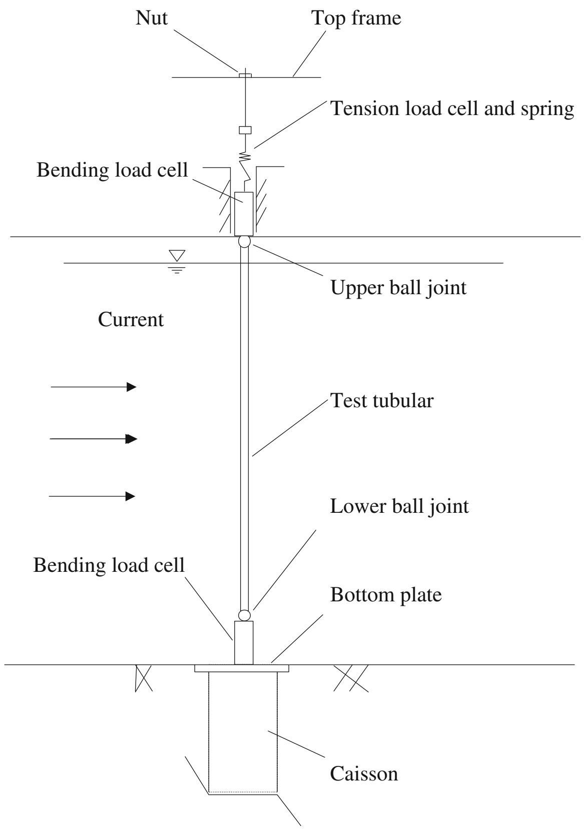

Fig. 2. Test set-up.

imposed at the center of the cylinder and was then released to obtain the dynamic motion. The motion signal was then acquired and processed to determine the frequency and damping. The mass properties, providing good prediction of the first frequency, were used to compute the second frequency. The tensions were those readings when the cylinders were in still water.

3. Test results and discussion

3.1. Vibration frequency

The measured cross-flow vibration frequencies of the two cylinders are displayed in Fig. 3. The horizontal axis is the reduced velocity, V/f_1D, based on the first in-water natural frequency of each cylinder. Two vertical lines are drawn in the plot to designate the primary lock-in zone within which there were significant VIV motion responses. One immediate observation from this plot is that the frequencies of the 4.5 in. cylinder, after an initial abrupt rise to a frequency about 10% higher than its natural frequency, increased only slightly (4.8%), while the frequencies for the other cylinder rose more than 25% with the reduced velocity (flow speed). This rather sudden change in the vibration frequencies of the 4.5 in. cylinder is a characteristic of lock-in: the motion of the structure takes control of the shedding process, such that the vortex shedding frequency shifts into the natural frequency of the cylinder. It happened quite noticeably for this bending-stiffness-dominated structure.

This disparity led us to the finding that it is the change in tension that was primarily responsible for the rise of the vibration frequency, as is explained below. A tensioned slender structure can often be classified into two types: beam-like and string-like. If the bending rigidity of a structure dominates tension, we call the structure a beam. If the tension dominates bending, we term it a string. This is characterized by a dimensionless parameter, TL^2/EI, where T is the tension, L is the cylinder length, E is Young's modulus, and I is the moment of inertia. The natural frequencies of such a

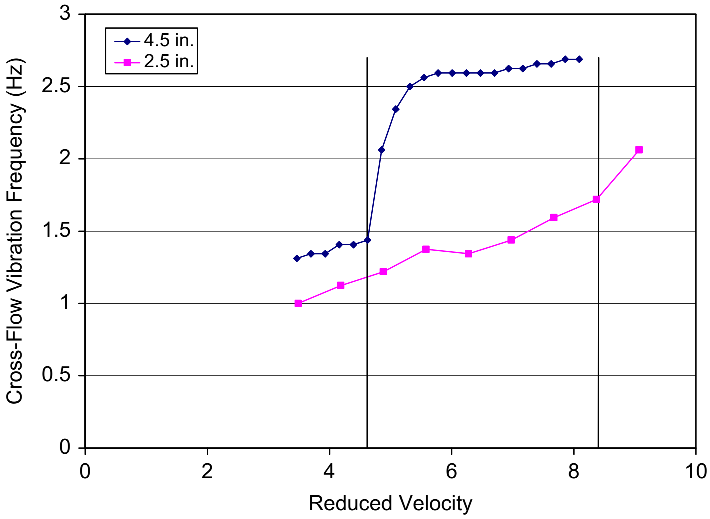

Fig. 3. Cross-flow vibration frequencies.

Table 2
Test cylinder characteristics and change in tension.

Test cylinder	4.5 in.	2.5 in.
TL^2/EI	0.36	3.20
Change in tension (%)	2.6	19.8

structure is given by (Clough and Penzien, 1975)

$$f_n = (n^2\pi/2L^2)\sqrt{(EI/m)}\sqrt{(1 + TL^2/n^2\pi 2EI)}, \quad n = 1, 2, 3, \ldots,$$

where m is the mass per unit length of the structure. As the flow speed increases, the steady (static) drag load on a cylinder rises, which tends to deflect the structure. This action is resisted by the stiffness of the structure. For a tension-dominated structure characterized by a larger TL^2/EI, this resistance comes primarily from the tension (it will rise once the flow speed increases), while for a bending rigidity controlled structure (with a smaller TL^2/EI), it is EI that provides such resistance. The term EI does not change with the flow speed/drag, but T will. The natural frequency will increase if T increases. This is the reason why the natural frequency and thus the vibration frequency of a tension-dominated structure will rise with flow speeds.

For these two cylinders, the values of TL^2/EI and the change in tension with flow speeds are provided in Table 2. Note that the value of T for computing TL^2/EI is the pre-tension listed in Table 1. The change in tension is based on the reduced velocity range from approximately 5.5 to 8.1 for each cylinder.

Clearly, the drag on the 4.5 in. cylinder was resisted primarily by its bending rigidity, EI. For the 2.5 in. pipe, the drag was opposed mainly by the tension, T. The top tension rose by approximately 20% with the flow speed for the 2.5 in. cylinder. The natural frequencies of a structure are in proportion to the square-root of the tension. This is the reason why the vibration frequencies of the 2.5 in. cylinders rose significantly with the flow speed, while that of the 4.5 in. cylinder did not. The square-root of the mean tension for both cylinders is plotted in Fig. 4.

Note that the key to the vibration frequency increase is the rise of tension, regardless of the magnitude of the applied tension. There are two scenarios when the axial tension in a cylinder can change significantly with flow speed. One is when the structural behavior of a cylinder is dominated by the applied tension rather than the bending rigidity (as discussed above); the other is when the axial stiffness is large (even if no tension is applied), such as the cases when both ends of the structure are constrained axially.

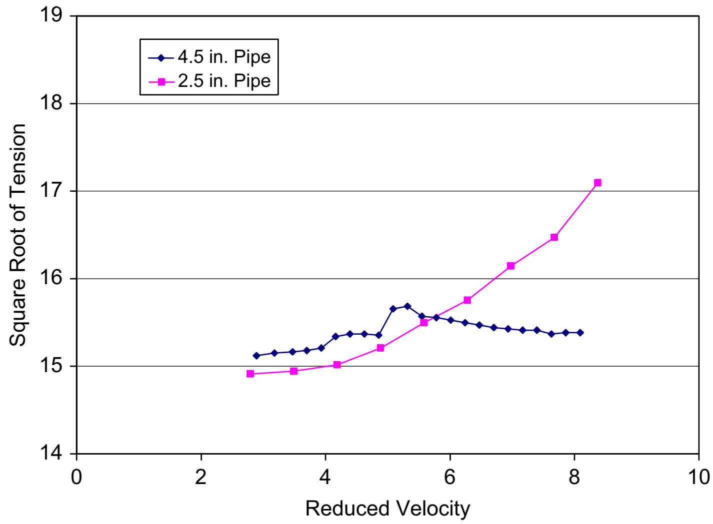

Fig. 4. Square-root of tension.

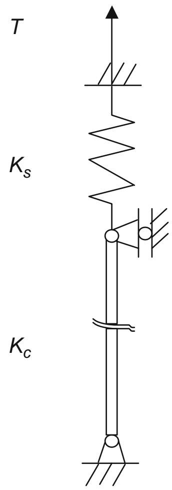

Fig. 5. Axial stiffness and tension.

If the axial stiffness is large, a change in the deflected shape will generate additional tension which in turn affects the natural frequencies. An additional test was performed to replace the top spring with a steel rod for the 4.5 in. cylinder, which greatly increased the axial stiffness of the cylinder (from 5.2×10^3 to 1.2×10^6 N/m). The tension changed by 46% for a speed range of 0.762–1.036 m/s, and resulted in an increase of vibration frequency by over 10%. Note that the axial load went beyond the design capacity of the load cells and the tests did not continue to higher speeds.

For the test set-up illustrated in Fig. 2, its structural model is sketched in Fig. 5. The total axial stiffness K_a can be written as

$$K_a = K_s K_c/(K_s + K_c),$$

where K_s is the stiffness of the spring and K_c the axial stiffness of the cylinder. If K_s approaches infinity (there is no added axial stiffness), then we have $K_a = K_c$.

Therefore, two parameters are identified as important to the change of tension: TL^2/EI and K_a. If either one of them is large, the natural frequency and thus the vibration frequency could rise significantly with flow speed.

The analysis in this section has been focused on vibration frequencies. Next, our attention will be shifted to the lock-in bandwidth, the reduced velocity range within which there is significant VIV motion.

3.2. Lock-in bandwidth

The VIV motion magnitudes of the 4.5 in. cylinder are displayed in Fig. 6 (Lee et al., 2004). The horizontal axis is the reduced velocity, V/f_1D, based on the first in-water natural frequency. While the vibration frequency did not change significantly, there existed a broad lock-in band for the 4.5 in. cylinder within which there was significant VIV motion. For example, as the flow speed increased such that V_r changed from 5.55 to 8.09 (a rise of approximately 46%), the vibration frequency rose from 2.562 to 2.688 Hz, only 4.8%. This is a rather small change, and it contradicts our common belief that it is the change of vibration frequency with flow speed (due to decrease of added mass) that constitutes the lock-in band. Clearly, there is a need for a new explanation. In the following, the authors provide a plausible one, based on what was often observed in laboratories (Sarpkaya, 1978).

It appears that, during lock-in, once a structure vibrates at its natural frequency and reaches at a sustained motion magnitude, the motion of the structure has control over the vortex-shedding process. The linear relationship between the flow speed and the vortex-shedding frequency, termed as the Strouhal relation for a fixed cylinder, is no longer valid. The structure has a tendency to stay with the vibration frequency, until it encounters a significant force with a very different frequency. This is a key difference from a mass-spring-dashpot (mechanical) resonator. The latter will always vibrate at the excitation frequency which is not dependent on the system itself. For a fluid–structure interaction system, the excitation forces, in terms of both magnitude and frequency, can be altered by the system responses and they tend to

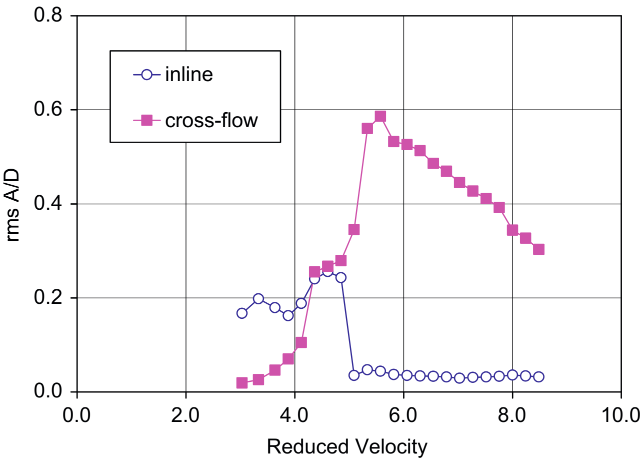

Fig. 6. VIV motion magnitudes of the 4.5 in cylinder.

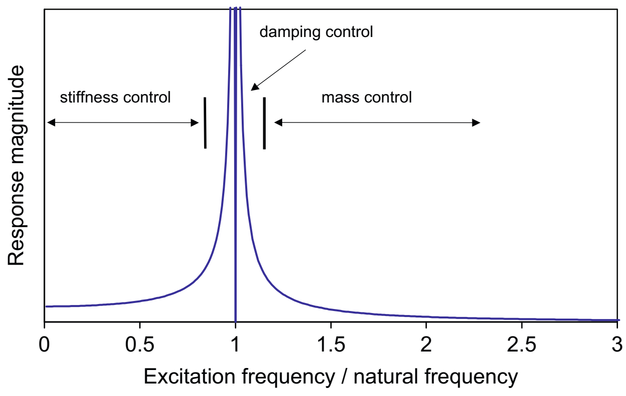

Fig. 7. Stiffness, mass, and damping controlled zones of a resonator.

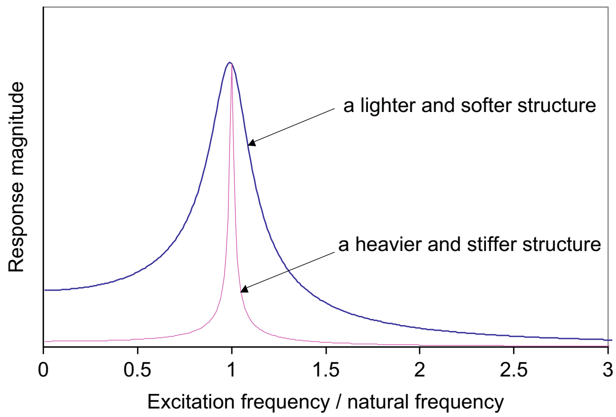

Fig. 8. Response bandwidth of a lighter/softer structure and a heavier/stiffer structure.

adapt to each other. It is the hydroelasticity that is at work. For the top-tensioned, flexible cylinders described herein, the effects of added mass do not appear to be as significant as commonly believed, even for the 4.5 in. cylinder with a mass ratio of approximately 1.0.

Questions remain on why a lighter/softer structure has a broader lock-in band than a heavier/stiffer one does. This can be explained with the aid of the resonance curve for a mechanical resonator, as depicted in Fig. 7. Which parameter governs the response magnitude will depend on what the ratio of the excitation frequency to the natural frequency is. If this ratio is smaller than 1 (outside the damping-controlled zone), it is the stiffness which matters: the stiffer a structure is, the smaller its motion response will be. If the frequency ratio is sufficiently greater than 1 (outside the damping controlled zone), the mass is more important. When the excitation frequency is close to the natural frequency, damping controls the motion magnitude.

The above description explains precisely why a lighter/softer structure has a broader lock-in band. As the excitation frequency is low, the stiffness controls the response. Since its stiffness is small, the response of this structure is large and that adds to the width of the lock-in band. Similarly, if the excitation frequency is greater than the natural frequency, the motion response is controlled by the mass. Since the mass is also small, the motion is large and that adds to the band on the other side. Overall, the lock-in bandwidth appears broader for a lighter/softer structure, as illustrated in Fig. 8. It is a characteristic of the fluid–structure interaction phenomenon that the shape of the resonance curve is more flat at the top and more abrupt, since the vortex-shedding frequency can suddenly shift to the natural frequency and stays with it (if the responses were plotted against the frequency ratio, one may only have discrete data points). It is further noted that:

(i) VIV is a self-limiting phenomenon: as the flow speed increases, the VIV motion of a cylinder rises to a certain level, and then the motion interferes with the vortex-shedding process and begins to break up the symmetric pattern of alternate vortices. The motion magnitude does not increase even if the flow speed continues to rise.
(ii) A fluid–structure interaction problem, such as VIV, is highly nonlinear. The resonant curve described in Fig. 7 is for a linear system. The response behavior of a cylinder experiencing VIV will no doubt deviate from this description. The absence of sharp peaks in the responses is likely such an example.

4. Conclusions and recommendations

Conclusions from this study are listed below:

(i) two parameters are identified as important to the change of tension: TL^2/EI and K_a. If either one of them is large, the vibration frequency of a cylinder could rise significantly with flow speeds;
(ii) the vibration frequencies of a bending stiffness-dominated cylinder do not rise significantly, but there is still a broad lock-in band, indicating a weak association between the change of the vibration frequency and the lock-in bandwidth;

(iii) the characteristics of the response curve of a mechanical resonator may be used to explain why a lighter/softer cylinder has a broader lock-in band than a heavier/stiffer one does.

Further tests should be performed to investigate the effects of added mass on the vibration frequency and bandwidth of a tensioned, flexible cylinder.

Acknowledgments

The authors would like to thank Shell for permission to publish this paper. We would also like to thank the following people: Dean Henning for his leadership in designing, fabricating, and conducting all of the cited experiments; Doug McMullen for computational support with both data processing and computational modelling, and Susan Viskovich for substantial assistance with manuscript preparation.

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